The Productive Power of Energy and its Taxation[†]

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Abstract
A modification of the neoclassical model of economic growth is based on the law of diminishing returns, which provides asymptotic boundary conditions for the differential equations the elasticities of production must satisfy. These elasticities give the weights with which small relative changes of the production factors contribute to the relative change of output. In this sense they measure the productive powers of capital, labor, and energy. The LINEX production function with factor- and time-dependent elasticities of production reproduces economic growth and the energy crises in Germany, Japan, and the USA since 1960 without Solow residual; it also maps the structural change at German reunification. The time-averaged elasticities of production turn out to be for labor much smaller and for energy much larger than the cost shares of these factors. A shift of taxes and levies from labor to energy according to the productive powers of these factors should reduce unemployment and contribute to emission mitigation. Alternatively, emission trading should be done with 100 percent auctioning of emission certificates.

Keywords: *Energy, entropy, economic growth, law of diminishing returns, elasticities of production, factor cost shares, energy taxation.*

1 Introduction: Energy and Entropy

Energy conversion and entropy production have significant impacts on industrial evolution and environmental stability. This is because – according to the First and the Second Law of Thermodynamics – nothing happens in the world without energy conversion and entropy production.

Quantitatively, the First Law states that *Energy = Exergy + Anergy* is a conserved quantity. Exergy is the valuable part of energy, which can be converted into physical work, whereas anergy is the useless part of energy, e.g. heat at temperature T_0 of the environment. The Second Law, in the general formulation of non-equilibrium thermodynamics, says that entropy-production density of non-equilibrium systems containing N different

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sorts of particles k is never negative and given by [1]

$$\sigma_{S,dis}(\vec{r},t) = \vec{j}_Q \vec{\nabla}(1/T) + \sum_{k=1}^{N} \vec{j}_k[-\vec{\nabla}(\mu_k/T) + \vec{f}_k/T] \geq 0 \quad . \tag{1}$$

In words: entropy production density in the space-time point $(\vec{r},t)$ consists of the heat current density $\vec{j}_Q$, driven by the gradient $(\vec{\nabla})$ of temperature T, and particle (diffusion) current densities $\vec{j}_k$, driven by gradients of chemical potentials μ_k and of temperature and by external forces $\vec{f}_k$. Whenever heat is generated, valuable exergy is converted into useless anergy. This is what "energy consumption" really means.

All processes of industrial production are irreversible so that $\sigma_{S,dis} > 0$. They are associated with emissions of particles, which may change the composition of the biosphere, and of heat, which change the energy flows through the biosphere. As long as heat emissions are considered as environmentally more benign than particle emissions, one can transform the latter into the former by appropriate technologies. The "heat equivalents of noxious substances" (HEONS) are a measure of the exergy consumed in the abatement of particle emissions and the resulting additional heat burden on the environment. Model calculations of HEONS in electricity generation have been done for SO_2, NO_x, CO_2, and nuclear waste [2]. Should present anthropogeneous heat emissions increase by about a factor of 30 and approach the so-called "heat barrier" of $3 \cdot 10^{14}$ Watts, climate changes are to be expected even without the anthropogeneous greenhouse effect.

There are economists who have realized that energy and entropy matter in economics. For instance, as early as 1927 Tryon stated: "Anything as important in industrial life as power deserves more attention than it has yet received from economists. . . . A theory of production that will really explain how wealth is produced must analyze the contribution of the element energy" [3], and in 1974 Binswanger and Ledergerber flatly declared: "The decisive mistake of traditional economics ... is the disregard of energy as a factor of production." [4] The economic importance of entropy was highlighted in 1971 by Georgescu-Roegen's seminal book "The Entropy Law and the Economic Process" [5], which stimulated research of ecological economists into the relevance of thermodynamics for economics [6, 7, 8].

On the other hand, traditional economics does not worry about the principal laws of thermodynamics, despite of their governing all energetic and material processes of industrial production. Of course, one realizes the problem of pollution and climate change [9], but does not believe in limits to growth in finite systems such as planet Earth. The option of expanding the economic system beyond the biosphere, e.g. by solar power satellites and space industrialization [10, 11, 12, 13], is taken seriously by few economists only. When it comes to the question, how wealth is produced, standard production theory usually takes only capital and labor into account. If, occasionally, energy is also considered as a factor of production, it is given a tiny economic weight only.

The marginal role attributed to energy in standard economic theory was very clearly described by the econometrician Denison. In a controversial discussion, whether the first oil price explosion 1973-1975 could have been related to the simultaneous world-wide recession, he argued: "Energy gets about 5 percent of the total input weight in the business sector . . . the value of primary energy used by nonresidential business can be put at \$ 42 billion in 1975, which was 4.6 percent of a \$ 916 billion nonresidential business national income. . . . If . . . the weight of energy is 5 percent, a 1-percent reduction in energy consumption with no change in labor and capital would reduce output by 0.05 percent."[14] Thus, the decrease of energy input in the US economy by 5.2 percent between 1973 and 1975 should have only caused a decrease of output by 0.26 percent. The observed decrease of output, however, was 1.0 percent. Thus, from this perspective the recessions of the energy crises, as shown in Figs. 1- 4, are hard to understand. The quoted input weight corresponds to the cost share of energy in total factor cost, which has been rougly 5 percent on an OECD average.

Cost-share weighting of production factors is the standard procedure in economic theory. However, in addition to the difficulties with explaining the energy crises, this factor weighting has the problem of the Solow residual. The "Solow residual" accounts for that part of output growth that cannot be explained by the weighted input growth rates. It amounts to more than 50 percent of total growth in many cases. Attributing this difference formally to technological progress "has lead to a criticism of the neoclassical model: it is a theory of growth that leaves the main factor in economic growth unexplained" [15].

Furthermore, there exists the widely-held belief in economics that there are nearly unlimited opportunities for substitution. Thus, the Nobel Laureate in Economics Robert M. Solow [16] expected that "The world can, in effect, get along without natural resources". Nevertheless he cautioned: ". . . if real output per unit of resource is effectively bounded . . . then catastrophe is unavoidable."[16] Since, because of the first two laws of thermodynamics, the output per unit of energy input is bounded indeed, the second part of Solow's statement represents a pessimistic vision of the future.

Independently from what the future may bring, let us look into the impact of energy on industrial growth in the past [17, 18]. In so doing we deviate from the path of traditional economics when it comes to factor weighting.

2 Modeling Industrial Production

The quantitative desription of industrial production is based on the following observations. 1. The elementary production processes are work performance and information processing. 2. The capital stock consists of energy-conversion devices and information processors, and all buildings and installations necessary for their protection and operation. 3. Labor manipulates capital. 4. Energy activates capital. 5. Creativity is the specific human contribution to economic evolution, which cannot be made by any machine capable of learning. It consists of ideas, inventions and value decisions.

The output (value added) Q, i.e. the gross domestic product (GDP) or parts thereof, is measured in constant currency, and so is the capital stock K. (In principle, both could be defined in terms of work performance and information processing [17, 18], but the empirical data are not available.) Routine labor L is measured in manhours worked per year, and primary energy E is measured in, e.g., Joules "consumed" per year. The empirical data on output and inputs are taken from the national accounts, labor statistics and energy balances. The effects of creativity can only be discovered ex post.

It is convenient to work with dimensionless variables, which change in time t. We define: $q(t) \equiv Q(t)/Q_0$ (normalized output), $k(t) \equiv K(t)/K_0$ (normalized capital stock), $l(t) \equiv L(t)/L_0$ (normalized labor), $e(t) \equiv E(t)/E_0$ (normalized energy input), where Q_0, K_0, L_0, E_0 are output and inputs in a base year t_0. Creativity causes an explicit time dependence of the *production function* $q = q(k, l, e; t)$ that is used to describe mathematically the growth of output.

Infinitesimal changes of output, dq, capital, dk, labor, de and time, dt are related to each other by the *growth equation* (which is obtained from the total differential of the production function):

$$\frac{\mathrm{d}q}{q} = \alpha\frac{\mathrm{d}k}{k} + \beta\frac{\mathrm{d}l}{l} + \gamma\frac{\mathrm{d}e}{e} + \delta\frac{\mathrm{d}t}{t-t_0} \quad . \tag{2}$$

The *elasticities of production*

$$\alpha(k,l,e) \equiv \frac{k}{q}\frac{\partial q}{\partial k}, \quad \beta(k,l,e) \equiv \frac{l}{q}\frac{\partial q}{\partial l}, \quad \gamma(k,l,e) \equiv \frac{e}{q}\frac{\partial q}{\partial e}, \quad \delta \equiv \frac{t-t_0}{q}\frac{\partial q}{\partial t} \tag{3}$$

give the weights, with which relative changes of the production factors k, l, e, and of time time t contribute to the relative change of output. In this sense they *measure the productive powers* of capital, labor, energy, and creativity.

We follow standard economics in assuming that production functions, at a fixed time t, are twice differentiable, lineary homogeneous state functions of the variables k, l, e within accessible factor space. This means that

$$\alpha + \beta + \gamma = 1, \tag{4}$$

and from the equality of the second-order mixed derivatives of $q = q(k, l, e; t)$ follow the differential equations

$$\begin{aligned} k\frac{\partial\alpha}{\partial k} + l\frac{\partial\alpha}{\partial l} + e\frac{\partial\alpha}{\partial e} &= 0, \\ k\frac{\partial\beta}{\partial k} + l\frac{\partial\beta}{\partial l} + e\frac{\partial\beta}{\partial e} &= 0, \\ l\frac{\partial\alpha}{\partial l} &= k\frac{\partial\beta}{\partial k}. \end{aligned} \tag{5}$$

The most general solutions of these equations are:

$$\alpha = A(l/k, e/k), \qquad \beta = \int \frac{l}{k}\frac{\partial A}{\partial l}\mathrm{d}k + J(l/e), \tag{6}$$

where $A(l/k, e/k)$, and $J(l/e)$ are any differentiable functions of their arguments $l/k, e/k$ and $l/e = (l/k)/(e/k)$.

Special solutions of the three coupled differential equations are i) the trivial solutions, i.e. the constants α_0, β_0, $\gamma_0 = 1 - \alpha_0 - \beta_0$. Standard cost-share weighting of economics uses $\alpha_0 \approx 0.25$, $\beta_0 \approx 0.70$, $\gamma_0 \approx 0.05$. ii) We use the simplest non-trivial solutions, satisfying asymptotic technical-economic boundary conditions. They are:

$$\alpha = a\frac{l+e}{k}, \quad \beta = a(c\frac{l}{e} - \frac{l}{k}), \quad \gamma = 1 - \alpha - \beta. \tag{7}$$

α satisfies the Law of Diminishing Returns: if the increase of the capital stock k exceeds by far the increse of labor l and energy e, an additional unit of k will contribute less and less to the growth of output. β results from α in the second of eqs. (6) and the condition that the weight of labor should approach zero for the factor combination that corresponds to the state of total automation. γ follows from eq. (4).

Inserting the elasticities of production into the growth equation (2) and integrating along any convenient path from the initial values to $(q; k, l, e)$ one obtains i) the energy-dependent Cobb-Douglas production function $q_{CDE} = q_0 k^{\alpha_0} l^{\beta_0} e^{\gamma_0}$, if one uses the trivial constants and ii) the LINEX production function

$$q_{Lt}(t) = q_0 e \exp\left[a(t)(2 - \frac{l+e}{k}) + a(t)c(t)(\frac{l}{e} - 1)\right], \tag{8}$$

if one uses the elasticities from eq. (7). The LINEX function depends *lin*early on energy and *ex*ponentially on quotients of capital, labor and energy. It contains the technology parameters a and c, which may become time dependent, if creativity acts. The capital-efficiency parameter $a(t)$ and the energy-demand parameter $c(t)$ are modeled by logistics or taylor series and are determined by non-linear OLS fitting of $q_{Lt}(t)$ to $q_{empirical}(t)$, subject to the constraints: $\alpha \geq 0, \beta \geq 0, \gamma \geq 0$.

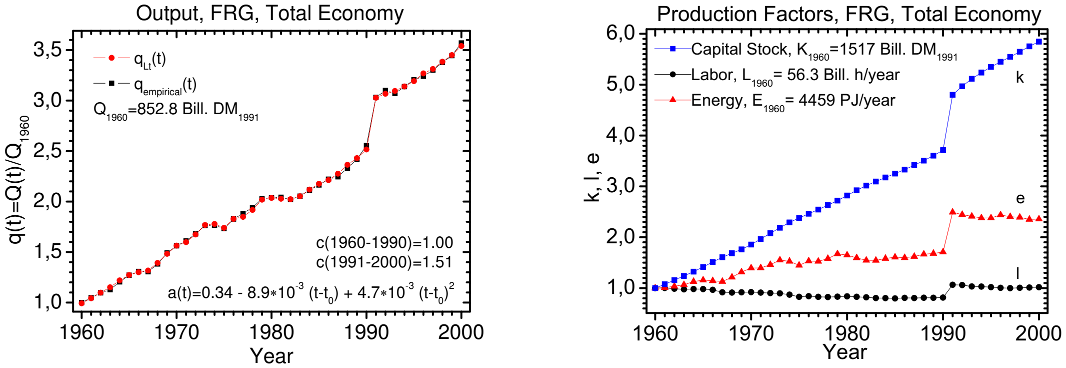

Figure 1: Left: Empirical growth (squares) and theoretical growth (circles) of the normalized output $q = Q/Q_{1960}$ of the total economy of the Federal Republic of Germany (FRG) between 1960 and 2000. Right: Empirical time series of the normalized factors capital $k = K/K_{1960}$, labor $l = L/L_{1960}$, and energy $e = E/E_{1960}$.

3 Economic growth in Germany, Japan and the USA

Figures 1 - 4 show the empirical time series of output $q_{empirical}$ and of the inputs k, l, e, and the theoretical output q_{Lt} calculated with the empirical inputs and the LINEX function for the total economy of the Federal Republic of Germany (FRG), the industrial sector "Warenproduzierendes Gewerbe" of the FRG, which produces about 50% of German GDP, the Japanese sector "Industries", which produces about 90% of Japanese GDP, and the total economy of the USA since 1960; the residential sectors are excluded. The Taylor-expansion model for $a(t)$ and $c(t)$ with a total of five free coefficients, used for computing q_{Lt} in Fig. 1, is indicated in this figure. The other growth curves are obtained with logistic functions modeling $a(t)$ and $c(t)$.[2] [19]

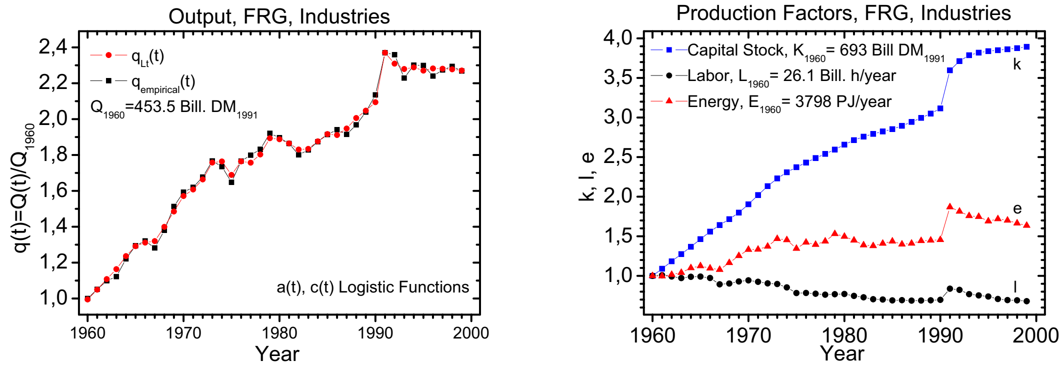

Figure 2: Left: Empirical growth (squares) and theoretical growth (circles) of the normalized output $q = Q/Q_{1960}$ of the German industrial sector "Warenproduzierendes Gewerbe" (GWG) between 1960 and 1999. Right: Empirical time series of the normalized factors capital $k = K/K_{1960}$, labor $l = L/L_{1960}$, and energy $e = E/E_{1960}$.

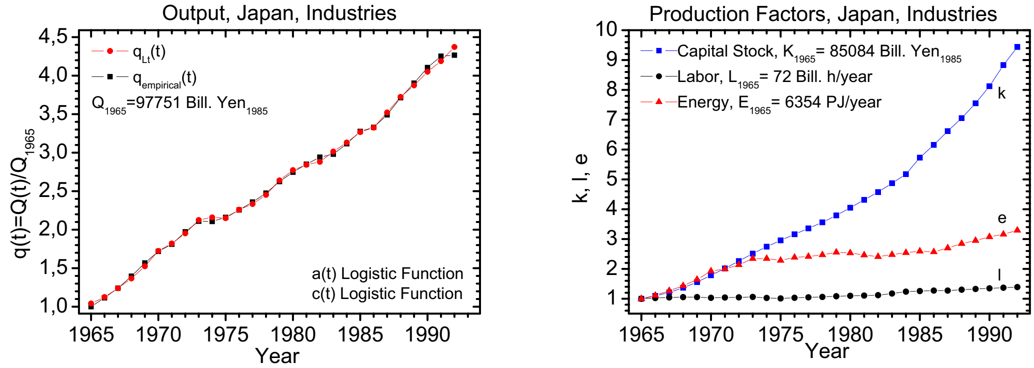

Figure 3: Left: Empirical growth (squares) and theoretical growth (circles) of the normalized output $q = Q/Q_{1965}$ of the Japanese sector "Industries" between 1965 and 1992. Right: Empirical time series of the normalized factors capital $k = K/K_{1965}$, labor $l = L/L_{1965}$, and energy $e = E/E_{1965}$.

In Figs. 1 – 4 the theoretical outputs q_{Lt} closely follow the empirical ones. They also reflect the ups and downs of the energy inputs during the energy crises 1973-1975 and 1979-1981. The sudden enlargement of the system "Federal Republic of Germany" (FRG) at reunification in 1990 is satisfactorily reproduced, too. The energy demand parameter

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[2] Logistics fit the total economy of the FRG even better, with $d_W = 1.94$. Otherwise, the results are quite similar to those of the five-coefficient model.

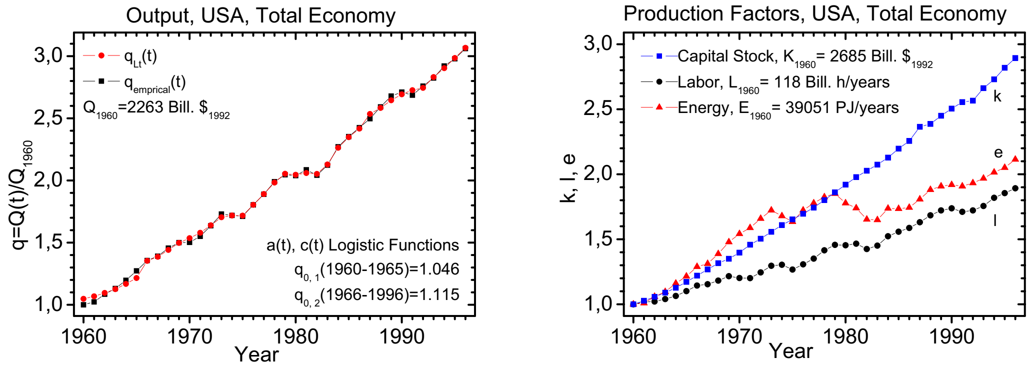

Figure 4: Left: Empirical growth (squares) and theoretical growth (circles) of the normalized output $q = Q/Q_{1960}$ of the total US economy between 1960 and 1996. Right: Empirical time series of the normalized factors capital $k = K/K_{1960}$, labor $l = L/L_{1960}$, and energy $e = E/E_{1960}$.

$c(t)$ rises steeply in 1991 for the German systems, reflecting that the energetically rather inefficient capital stock of the former German Democratic Republic was added to the capital stock of the Federal Republic of Germany.

4 Productive Powers of Capital, Labor and Energy

The time-dependent technology parameters $a(t)$ and $c(t)$ and the empirical values of $k(t)$, $l(t)$ and $e(t)$ are inserted into eq. (7). Then the time averages of these LINEX elasticities of production are computed. The results are shown in Table 1.

Table 1 Average LINEX elasticities of production (productive powers) of capital ($\bar{\alpha}$), labor ($\bar{\beta}$), energy ($\bar{\gamma}$), and creativity ($\bar{\delta}$), adjusted coefficient of determination R^2 and Durbin-Watson coefficient d_W for the Federal Republic of Germany's total economy (FRG TE) and its industrial sector "Warenproduzierendes Gewerbe" (FRG I), the Japanese sector "Industries" (Japan I), and the total economy of the USA (USA TE) during the indicated time spans.

System	FRG TE 1960-2000	FRG I 1960-99	Japan I 1965-92	USA TE 1960-96
$\bar{\alpha}$	0.38±0.09	0.37± 0.09	0.18± 0.07	0.51± 0.15
$\bar{\beta}$	0.15±0.05	0.11±0.07	0.09±0.09	0.14±0.14
$\bar{\gamma}$	0.47±0.1	0.52±0.09	0.73±0.16	0.35±0.11
$\bar{\delta}$	0.19±0.2	0.12* ± 0.13	0.14±0.19	0.10±0.17
R^2	1	0.996	0.999	0.999
d_W	1.64	1.9	1.71	1.46

The Durbin-Watson coefficients d_W, which are rather close to their best value 2, indicate that there is not much autocorrelation left.[3]

Comparing the average elasticities of production with the shares of the production factors in total factor cost, which are roughly 25% for capital, 70% for labor, and 5% for energy in highly industrialized countries, we note that $\bar{\beta}$ is much smaller and $\bar{\gamma}$ is much larger than the respective cost shares of labor and energy.

Ayres and Warr describe US economic growth from 1900 to 1998 with constant technology parameters and small residuals, using the LINEX function and exergy data [20]. Their average elasticities of production are similar to ours.

5 Conclusions: Energy Taxation

We see that energy is cheap and has a high productive power, while labor is expensive and has a low productive power. This results in the pressure to increase automation, substituting cheap energy/capital combinations for expensive labor. It also reinforces the trend towards globalization, because goods and services produced in low-wage countries can be transported cheaply to high-wage countries. The consequences are: Routine jobs get lost in high-wage countries and profits increase for the owners of the energy sources and the masters of the energy conversion devices. In addition, climate-destabilizing emissions grow with the increasing use of fossil energies.

Therefore, in order to fight increasing unemployment (and state indebtedness) and stimulate energy conservation and emission mitigation, the disequilibrium between the productive powers and cost shares of labor and energy should be reduced by shifting the burden of taxes and levies from labor to energy so that these factors' cost shares come closer to the factors' productive powers. This would mean that the tax and levy shares should be for labor 10-20%, capital 30-40% and energy 40-50% of the total tax and levy burden. In order to keep total revenues at the constant level accepted by society, the tax per energy unit should increase according to progress in energy conservation. Border tax adjustments according to the energy required for production and transportation of the border-crossing goods prevent competitive disadvantages in relation to not-energy-taxing countries. No recessions like the ones due to the oil price shocks are to be expected, because the wealth created by energy is not transferred abroad but only redistributed within the country.

In the 1990s the Commission of the European Union has proposed several energy taxation schemes. However, it turned out that emission trading is politically preferred to energy taxes. It may have social benefits, too, if all emission certificates are auctioned – just as properties of the vacuum were auctioned for the UMTS licences, temporarily relieving some state budget problems. But under the present EU grandfathering regime, where at most 10% of the emission certificates may be auctioned, the market price of the given-away emission certificates is charged as opportunity costs to the customers of the utilities. This has generated huge utility profits. Furthermore, at small trading volumes, oligopolies

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[3] The asterisk at the value 0.12 of $\bar{\delta}$ for FRG I indicates that the very large derivative of the logistic function $c(t)$ in 1991 has been omitted when calculating the time average. This has been also done for FRG TE, because the derivative of the step function $c(t)$ does not exist in 1991.

may easily manipulate the market price of the certificates. Should careful measurements of actual emissions be required for emission trading, the costs may be higher than the measurement costs for energy taxes.

A careful assessment of all economic instruments for preserving social and climate stability is necessary.

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